This analytic investigation (see Slepian [4] and Heurtley [5]) involves the derivation of a differential equation (using the method of commuting operators) for the functions $S_{\alpha,n}(c, x)$,[1] viz.,

$$(1-x^2)S_{\alpha,n}''(c,x)+\left(\frac{1}{x}-3x\right)S_{\alpha,n}'(c,x)+\left(\Gamma_{\alpha,n}(c)-\frac{3}{4}-c^2x^2-\frac{\alpha^2}{x^2}\right)S_{\alpha,n}(c,x)=0. \quad (2)$$

Here the differential equation eigenvalue $\Gamma_{\alpha,n}(c)$ is determined by requiring that $S_{\alpha,n}$ be finite for $x=0$, 1. Using the solutions of the differential equation Slepian [4] obtains the following results for $\gamma_{\alpha,n}(c)$.

1) Fixed α, n. Small c.

$$\gamma_{\alpha,n}(c)=\frac{(-1)^n\Gamma(n+1)\Gamma(n+\alpha+1)c^{2n+\alpha+1}}{2^{2n+\alpha+1}\Gamma(2n+\alpha+1)\Gamma(2n+\alpha+2)}\cdot\left[1-\frac{(2n+\alpha+1)\alpha^2c^2}{4(2n+\alpha)^2(2n+\alpha+2)^2}+0(c^4)\right]^2. \quad (3)$$

2) Fixed α, n. Asymptotically large c.

$$\gamma_{\alpha,n}(c)=(-1)^n\left\{1-\frac{\pi 2^{2\alpha+4n+2}c^{2n+\alpha+1}e^{-2c}}{\Gamma(n+1)\Gamma(n+\alpha+1)}[1+0(c^{-1})]\right\}. \quad (4)$$

He has also considered the case of fixed α and asymptotically large n and c. In (3) and (4) the parameter c is given by

$$c=\frac{kR^2}{2z_0}, \quad (5)$$

where k is the wavenumber, R is the radius of the circular reflector or lens, and $2z_0$ is equal to the reflector or lens separation; z_0 is their focal length. The power loss in decibel per iteration is given by

$$20\log_{10}[|\gamma_{\alpha,n}(c)|]; \quad (6)$$

in [1] and [3] the authors used the parameter a instead given by

$$a=\left(\frac{k}{2z_0}\right)^{1/2}R=c^{1/2}.$$

Losses calculated using (3) and (4) agree well with those given in references 1 and 3, as well as McCumber's [6] recent tabulations.

Similar results for rectangular mirrors or lenses have also been given by Slepian [7].

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[1] The $S_{\alpha,n}$ are called "generalized prolate Spheroidal functions" by Slepian and "hyperspheroidal functions" by Heurtley.
[2] Note the error in equation (97) of Slepian [4]; the values listed in Table I of [4] are correct.

Ferromagnetic Resonance Linewidth and *g*-Factor in Ferrites from 2 to 18 Gc/s

The parameters of the uniform precession in homogeneously biased ferrite spheres such as the linewidth and the g-factor are often used to characterize ferrite materials. As is well known these data are not always frequency independent. When they are measured using the standard cavity technique it is very difficult to obtain information over a broad frequency band. On the other hand, it is the actual frequency dependence that may be of interest both for microwave applications and for a theoretical understanding of the loss mechanism in ferrites.

The purpose of this correspondence is to present room-temperature measurements on spheres of some polycrystalline ferrites and yttrium iron garnet (YIG) obtained by the crossguide coupler technique proposed by Stinson [1]. X-band and Ku-band waveguide couplers with standard cross sections were used. The diameter of the coupling holes were 4 mm and 3 mm, respectively. For the lower band from 2 to 8.2 Gc/s a coaxial coupler [2] was constructed from a (3.0 mm/6.5 mm) coaxial line with a coupling hole of 4-mm diameter. The wall thickness of the coupling hole was in all cases about 0.3 mm. The ferrite spheres with a diameter of 1 mm throughout all measurements were fixed in the guides by polyfoam slabs.

In contrast to Stinson's original arrangement also, the secondary guide was short-circuited at one port in a distance of a half-guide wavelength from the coupling hole. Hence, the power coupled to the matched detector is increased by 6 dB in contrast to the case when the two arms of the secondary guide are matched. This may be important for measurements on broad linewidth ferrites of small volume. In addition, we have calculated the influence of the radiation damping of the primary and the secondary waveguide provided the coupler is excited by a matched source. The power coupling from the primary to the secondary waveguide with a short located a multiple of a half-waveguide wavelength from the center of the coupling hole can than be written as

$$\frac{C}{dB}=20\log\left|K_1\frac{4\pi^2r^3}{3ab\lambda_g}\right|+20\log\left|\frac{\chi_{xy}}{1+K_2j\dfrac{4\pi^2r^3}{3ab\lambda_g}\chi_{xx}}\right| \quad (1)$$

where a and b are the waveguide wide and narrow dimensions, respectively, r is the radius of the ferrite sphere, λ_g the guide wavelength, and χ_{xx} and χ_{xy} are the diagonal and off-diagonal magnetic susceptibility elements, respectively, defined in terms of the external microwave magnetic field. The quantities K_1 and K_2 depend on the circuitry of the secondary guide. When the two secondary arms are matched $K_1=1$ and $K_2=1.5$; and when one arm is matched and the other arm shorted at $\lambda_g/2$ from the hole, $K_1=2$ and $K_2=2$. If the radiation damping is not regarded, $K_2=0$ and we obtain Stinson's formula with $K_1=1$. The calculation is based on the assumption that the diameter of the sphere is much smaller than the diameter of the hole and that the wall thickness is much smaller than the diameter of the sphere. The ferrite linewidth ΔH_m measured at constant frequency and obtained from the difference of the two dc magnetic field strengths at the 3-dB points then follows from (1) as

$$\Delta H_m=\Delta H+K_2\frac{4\pi^2r^3}{3ab\lambda_g}M_s. \quad (2)$$

Here ΔH is the linewidth defined as the difference H_2-H_1 for

$$|\chi_{xy}(H_{1,2})|^2=0.5|\chi_{xy}|^2_{\max},$$

and M_s is the saturation magnetization of the ferrite. The influence of the radiation damping can be neglected at broad linewidth materials and $r<1$ mm. With single crystal garnets at $\Delta H\approx 0.5$ A/cm, however, the error can become large in the order of one-hundred per cent at usual dimensions and frequencies [3]. An additional shift of the resonance by the radiation damping however can be neglected in all these cases.

The frequency dependence of the linewidth ΔH obtained in this manner for some polycrystalline materials (R1, R5, and R6 from General Ceramics, YIG from Microwave Chemicals Lab., both U.S.A., and FXC4B and FXC5E1 from Philips, Germany) is shown in Fig. 1.

The behavior of the YIG linewidth is characterized by the sharp peak at 4 Gc/s where the uniform precession enters the spinwave manifold, in accordance with measurements first reported by Buffler [4]. The very high linewidth values of R1, FXC5E1, and FXC4B at low frequencies can be understood for these ferrites are no longer saturated at the corresponding resonance fields. Besides, it is remarkable that for all materials the linewidth of the uniform precession inside the spinwave manifold increases more or less with increasing frequencies as can be expected from theory.

When the resonance frequency of the uniform precession in a sphere is written in the

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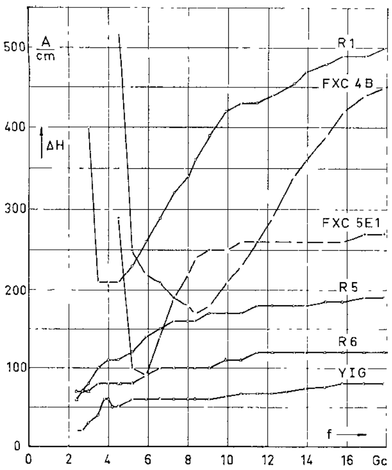

Fig. 1. Linewidth ΔH vs. frequency of polycrystalline spheres, 1-mm diameter.

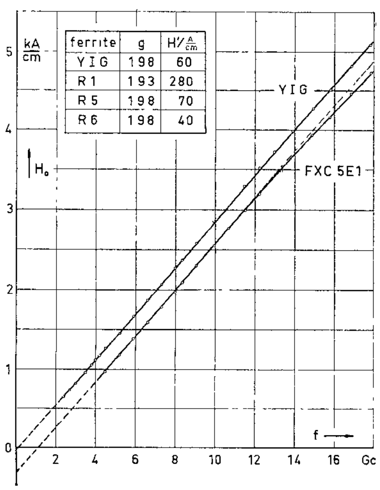

Fig. 2. Resonance field H_0 vs. frequency for FXC5E1 and YIG.

well-known form suggested by Okamura, et al. [5], the g-factor can be regarded as a value independant of frequency. Measurements at the R-ferrites and YIG have proved this statement. The g-factors of these materials are tabulated in Fig. 2 as well as the additional field H' introduced by Okamura, et al. The resonance behavior of FXC5E1 at room temperature, however, seems to be anomalous at frequencies above 12 Gc/s as can be seen from the diagram Fig. 2which gives a plot of the resonance field vs. frequency of this ferrite and, comparatively, that of YIG. An explanation of this behavior which has already been observed [4], [6] on FXC4B and other ferrites can not be given.

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International Intercomparison of Standards for Microwave Power Measurement

In accordance with a resolution of Commission I of the International Scientific Radio Union (URSI), standards for microwave power measurement of Japan, the United Kingdom (England) and the United States have been intercompared. It is the purpose of this correspondence to describe the results and comment upon the significance of these intercomparisons.

The interest of URSI in microwave power measurements extends back to the VIIth General Assembly which was held in Paris in 1946, and the resolution recommending the intercomparison of microwave power measuring techniques, which originated at that meeting, has been reaffirmed by each succeeding General Assembly. The potential usefulness of intercomparisons between the Primary Standards Laboratories of various countries was thus recognized well over a decade ago. It has only been in the past few years, however, that this resolution has been implemented.

Although the United Kingdom had, at an earlier date, affirmed its readiness and willingness to participate in such intercomparisons, to the Japanese National Committee also goes a large measure of credit for initiating this program when their delegates to the XIIth General Assembly (Boulder, Colorado, 1957) brought with them a calibrated bolometer mount and requested that it be compared with similar standards at the National Bureau of Standards, Boulder Laboratories (which were then in an advanced stage of development).

From this beginning, a total of five intercomparisons have been completed between Japan and the United States, two between Japan and the United Kingdom, and two between the United States and the United Kingdom. In each case the transfer standard employed has been a calibrated bolometer mount. For the purpose of reporting, the material may be conveniently divided into two sections:

1) Intercomparisons between Japan and the United States; and
2) The United Kingdom/Japan, United Kingdom/United States intercomparisons.

The details follow.

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Comparisons between Japan and the U. S.

The microwave power measurement techniques employed by both Japan and the United States are based upon refinements [1], [2] in the original work of Macpherson and Kerns [3]. The essential features of this technique include a microcalorimeter which is so devised as to permit a simultaneous calorimetric and bolometric determination of the power dissipated in a waveguide bolometer mount. The difference between the two measurements is attributed to the dc-RF substitution error and to power dissipation in the bolometer mount other than in the bolometer element (mount inefficiency). These two effects encompass the major sources of systematic error associated with the bolometric technique. In this way an effective efficiency (η_e) is obtained and employed in subsequent bolometric measurements. The bolometer mount may then be employed to determine the effective efficiency of other bolometer mounts by using well-known techniques [4]–[6].

In a number of these intercomparisons the bolometer mounts were also evaluated by the Kerns impedance method [7]. This procedure yields only the *efficiency* (η) of the bolometer mount in contrast with the *effective efficiency* (combined effect of efficiency and substitution error) as determined by the microcalorimetric method. Experience to date would tend to suggest, however, that the substitution error is much the less important of the two errors, at least at frequencies of 10 GH_z and lower. In addition the impedance method requires rather complex instrumentation and exacting attendant procedures, and is based upon a postulate whose validity is difficult to satisfactorily establish. For these reasons a somewhat greater level of confidence is assigned to the calorimetric determinations, although the results of these intercomparisons would appear to imply that a high order of accuracy is also possible with the impedance method. The procedure employed in Japan was based on a modification of the Beatty-Reggia [8] version, whereas in the United States a more recently developed [9] variation of the technique is now in use.

The first intercomparison was effected by means of a bolometer mount provided by Japan and designated J9-1. The results were as follows:

J9-1

Japan	United States
July 1957 $\eta_e = 96.52 \pm 0.5\%$ $\eta = 96.03 \pm 2\%$ January 1958 $\eta_e = 09.36 \pm 1.9\%$ $\eta = 90.01 \pm 2\%$	December 1957 $\eta_e = 90.6 \pm 0.8\%$